Optimization of optical coatings in optical systems

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ABSTRACT

The spectral response of a non-uniform optical coating in a particular optical system is analyzed. The optical coating in the optical system is optimized having as targets the spectral responses (integral reflection factors) of the coating deposited on a diopter in a particular optical system. A conversation is established between the designing program for optical coatings and the designing program for optical systems. The designing program for optical coatings communicates to the designing program for optical systems the coating and the targets to be optimized. The designing program for optical systems evaluates the merit function and communicates it to the designing program for optical coatings, which accordingly modifies the optical coating parameters to attain an optimum, depending on the merit function evolution. The process is repeated until the merit function reaches an extreme.

Keywords: Coatings, Thin films, Optical systems

1. Introduction

When designing optical systems we are confronted with the following problem: we have at our disposal a set of optical coatings with determined spectral functions - generally characterized for incidences and polarization states of given incident light on plane-parallel supports with specified indexes of refraction - which are to be used in a particular optical system on the diopter surfaces. Optical coatings have a well-established manufacturing technology, with a given evaporation geometry, by means of which, the optical coatings are deposited on the different-shaped surfaces of the optical components. It is important to find whether the optical coatings chosen for the diopter surfaces in the designed optical system and which are manufactured by pre-established manufacturing technologies have the expected spectral response (reflection, transmission), i.e. whether the optical coating accomplishes the function for which it was designed. The modification of the spectral response of an optical coating is due to the various incidence and polarization states of the light rays on the optical coating, generally, different from the situation considered at the design stage. Also, when producing optical coatings on the diopter surfaces, there may result non-uniform coatings with errors in terms of the geometrical thickness and the refraction indexes of the coating layers. It is consequently advisable to analyze the spectral response of an optical system coating from the very design stage of that system. When the spectral response is not as expected, the optical coating should be either replaced or redesigned (the manufacturing technology included) in order to get the optimum spectral response for that particular case.

The spectral response of an optical coating is determined with the designing program for optical systems. Since the spectral response of a coating depends on the polarization state of the incident light, it is obligatory the polarization ray tracing should also accompany that ray tracing through the system. Polarization ray tracing is presented at length in Refs. 1, 2, 3 therefore the subject will not be addressed in this paper. Most market-available designing programs for optical systems have polarization ray tracing where the presence of optical coatings is also considered.

As mentioned above, the optical coatings on the surfaces of the optical components could be non-uniform and such non-uniformity is determined by the evaporation geometry, as well as, by the shape and size of the surface. Optical coating uniformity is discussed in Refs. 4, 5, 6, 7. In these papers uniformity is analyzed on the workholders on which the optical components are placed during the manufacturing process. In order to increase the uniformity on plane, spherical, conical and pyramidal workholders, uniformity screens[8] either stationary or movable (turning around the symmetry axis of the workholder) can be designed. Another way of increasing uniformity is to use the geometry of a planetary system where the optical component is subject to a movement of rotation and revolution. The fact must be stressed that a high uniformity obtained on various types of workholders does not necessarily mean a high uniformity on the surfaces of the optical components placed on such workholders. The same applies to the geometry of a planetary system. The geometry of a planetary system has plane, spherical, conical and pyramidal geometries, as particular cases. The possibility to numerically determine the uniformity in the geometry of a planetary system[9] allows to determine the uniformity on the surfaces of the optical components with all types of evaporation geometries. The uniformity determined this way could be used in evaluating the spectral response of optical coatings in optical systems, which makes the subject of this paper.

2. Tracing Polarization Ellipse Parameters

Unlike the geometrical ray tracing through optical systems, in polarization ray tracing, the ray is also characterized by the polarization state and the total intensity I_t at a specified wavelength. The polarization state is described by the amplitudes E_ξ and E_η of the electric field vector components in two perpendicular directions (O_ξ, O_η), (which are normal to the direction of propagation), the orientation of the components $\mathbf{E}_\xi$ and $\mathbf{E}_\eta$, the phase difference δ between the two components and the degree of polarization P, where $P = I_{polarized}/I_t$. The total intensity is the sum of the unpolarized and polarized intensities:

$$I_t = I_{polarized} + I_{unpolarized}, \quad \text{where } I_{polarized} = E_\xi^2 + E_\eta^2 \tag{1}$$

The optical coating changes the polarization state and the total intensity of the incident rays. For a ray with an incidence different from the normal one, the optical coating has as reflection and transmission factors $\mathbf{R}_s$, $\mathbf{R}_p$, $\mathbf{T}_s$, $\mathbf{T}_p$ as well as, phase changes in reflection φ_s^r , φ_p^r and transmission φ_s^t, φ_p^t, where **s** and **p** are the directions normal and parallel to the plane of incidence. There must be determined how reflection and transmission change the polarization state and the total intensity of an incident ray on the optical coating. That can be done using the Jones matrix, Muller matrix or the polarization ellipse parameters. In this paper the last method is discussed. The parameters describing the polarization ellipse for an incident ray are: the amplitudes E_ξ and E_η, orientation of the components $\mathbf{E}_\xi$ and $\mathbf{E}_\eta$, the phase difference between the components $\mathbf{E}_\xi$ and $\mathbf{E}_\eta$ ($\mathbf{E}_\xi$ and $\mathbf{E}_\eta$ are coherent), the angle ψ between the major axis of ellipse related to one of the normal directions. When the phase difference between $\mathbf{E}_\xi$ and $\mathbf{E}_\eta$ is $\delta = \pi/2$, the ellipse axes are coincident with the directions $\mathbf{E}_\xi$ and $\mathbf{E}_\eta$, the latter being the ellipse semi-axes. We assume that the polarization ellipse of the incidental ray has $\delta = \pi/2$ (ellipse axes are coinciding with the directions $\mathbf{E}_\xi$ and $\mathbf{E}_\eta$, Fig. 1). Generally, the directions $\mathbf{E}_\xi$ and $\mathbf{E}_\eta$ do not coincide with the direction **s** and **p** describing the plane of incidence.

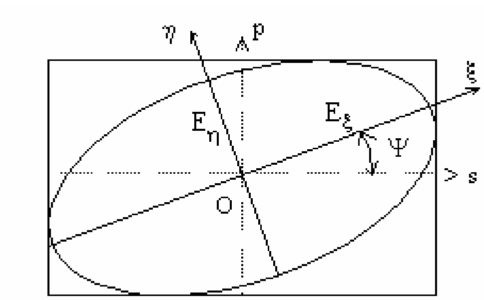

Fig. 1 Polarization ellipse representation

The vectors **s** and **p** are rotated respect of $\mathbf{E}_\xi$ and $\mathbf{E}_\eta$ by an angle ψ. The polarization ellipse in the system of axes **s** and **p** can be expressed by means of $\mathbf{E}_s$, $\mathbf{E}_p$ and δ, using the relations[10]:

$$\pm E_\xi E_\eta = E_s E_p \sin\delta \tag{2}$$

$$E_\xi^2 + E_\eta^2 = E_s^2 + E_p^2 \tag{3}$$

$$\tan 2\Psi = 2\frac{E_s E_p}{E_s^2 - E_p^2}\cos\delta \tag{4}$$

The parameters of the incident ray E_s, E_p and δ, are changed by reflection or transmission on the optical coating. The polarization state of the reflected or transmitted ray is expressed according to the directions **s** and **p**, related to which the major axis of the ellipse can be rotated by an angle Ψ'. New directions $\mathbf{E}_\xi$ and $\mathbf{E}_\eta$ are searched for using the relations (2, 3,

4) for which the polarization ellipse of reflected or transmitted rays should have $\delta = \pi/2$. It is thus possible to pass to the next diopter.

The amplitudes of the components $\mathbf{E}_s$ and $\mathbf{E}_p$ reflected and transmitted for $I_{polarized}$ are:

Reflection:

$$E_s^r = \sqrt{R_s} E_s^i, \qquad E_p^r = \sqrt{R_P} E_p^i \tag{5}$$

The phase difference δ is changed with $\varphi_s^r - \varphi_p^r$

Transmission:

$$E_s^t = \sqrt{T_s} E_s^i, \qquad E_p^t = \sqrt{T_P} E_p^i \tag{6}$$

The phase difference δ is changed with $\varphi_s^t - \varphi_p^t$

If the polarization degree of the incident ray $P < 1$ (there is unpolarized light) and the spectral response for the directions **s** and **p** is different, then from the unpolarized intensity it appears a polarized component. The unpolarized light can be considered to be a superposition of two incoherent linearly polarized components of equal intensities. For instance, if we assume $R_s > R_p$, then in reflection occurs a polarized component **s**:

$$I_s^{(pol)} = (R_s - R_p) I_{unpolarized} / 2 \tag{7}$$

with

$$E_s^{(pol)} = \sqrt{I_s^{(pol)}} \tag{8}$$

The polarized component $E_s^{(pol)}$ is not coherent with the component E_p^r determined by relation (5). Having $I_p^r = (E_p^r)^2$ and assuming for instance that $I_p^r \geq I_s^{(pol)}$, we can write

$$I_p^r = I_p^{'r} + I_s^{(pol)} \tag{9}$$

As the components E_p^r (only $I_s^{(pol)}$) and $E_s^{(pol)}$ are incoherent, they describe an unpolarized component. The reflected unpolarized intensity is:

$$I_{unpolarized}^r = R_P I_{unpolarized} + 2I_S^{(pol)} = (R_s + R_p) I_{unpolarized} / 2 + I_s^{(pol)} \tag{10}$$

The component **p**, reflected and coherent with E_s^r is

$$E_p^{'r} = \sqrt{I_p^r - I_s^{(pol)}} = \sqrt{\left(E_p^r\right)^2 - I_s^{(pol)}} \tag{11}$$

The total reflected polarized intensity is

$$I_{polarized}^r = \left(E_s^r\right)^2 + \left(E_p^{'r}\right)^2 \tag{12}$$

The total reflected intensity is

$$I_t^r = I_{polarized}^r + I_{unpolarized}^r \tag{13}$$

while the degree of polarization of the reflected ray is

$$P^r = I_{polarized}^r / I_t^r \tag{14}$$

If $I_p^r < I_s^{(pol)}$ then in reflection we only have the polarized component **s**. Similar computation can also be done for transmission. The algorithm should describe the case of the light transmitted by two plane-parallel plates, with unpolarized incident light and at Brewster incidence, with the planes of incidence orthogonaly oriented. The light transmitted by the first plate is partially polarized while the light transmitted by the second plate is unpolarized.

3. Integral reflection and transmission factors of optical coatings in optical systems

Let us assume an optical system where an optical coating is produced on the surface of diopter *i*. From an object point P_0 *n* rays are generated[10], with total intensities I^o_j ($I_{total} = I_{polarized} + I_{unpolarized}$), $j = 1, 2, \ldots n$, and specified states of polarization, passing through the entrance pupil. On diopter *i*, *m* rays are incident, with $m \leq n$, with total intensities I^{io}_j, $j = 1$, 2, …m. By reflection or transmission on diopter i there result reflected or transmitted rays with total intensities I^{ir}_j, I^{it}_j, j = 1,2,…m. The integral reflection and transmission factors for coating on diopter i, at one wavelength done, are:

$$R_i = \frac{\sum_{j=1}^{m} I_j^{ir}}{\sum_{j=1}^{m} I_j^{i0}}, \qquad \mathrm{T_i} = \frac{\sum_{j=1}^{m} I_j^{it}}{\sum_{j=1}^{m} I_j^{i0}} \tag{15}$$

If we have:

$$\sum_{j=0}^{m} I_j^{i0} = 0 \tag{16}$$

R_i, T_i = 0 (there is no reflected or transmitted light). This case occurs when, at a wavelength, the intensity is completely absorbed by the (selectively absorbtion) optical materials of the system blocked by the optical coatings preceding diopter i or vignetted. When we have a selective absorption by the optical materials of the system or the optical coatings on the diopters preceding diopter i are selective in the working spectral range of the analyzed optical coating, it is important to know the spectral composition of the incident light on the coating. The analysis of the spectral composition of the incident light on a coating can lead to the use of some simpler coatings with greater tolerances. We define the average transmitted and reflected intensities on diopter i, at a wavelength, as follows:

$$I_{iR} = \frac{\sum_{j=1}^{m} I_j^{ir}}{m} \qquad \mathrm{I_{iT}} = \frac{\sum_{j=1}^{m} I_j^{it}}{m} \tag{17}$$

If rays are generated with I^o_j = 1, then I_{iR}, I_{iT} coul be compared to R_i, T_i. The transmission factor of the first diopters i in the optical system, at the done wavelength, is defined as follows:

$$T = \frac{\sum_{j=1}^{m} I_j^{it}}{\sum_{j=1}^{n} I_j^{0}} \tag{18}$$

As mentioned above, the optical coating deposited on the diopter i surface, in the evaporation geometry defined in the manufacturing technology, can be non-uniform. Most of the optical components of optical systems have rotation symmetry surfaces (sphere, paraboloid, ellipsoid etc). In most cases non-uniformity of the coatings deposited on these surfaces is not symmetrical to the axis of symmetry. However, for simplicity reasons, hereinafter it will be considered symmetrical and described by the sag. All the points belonging to the intersection between a plane normal to the axis of symmetry (positioned in relation to the diopter point by the sag) and the diopter surface, have the same uniformity. In most cases, the manufacture of optical coatings is photometrically[11] controlled, by measuring the reflection and transmission factors of the coating, or of a part of the coating, deposited on a fixed test plate. For this reason the coating uniformity will be further described by the geometrical coefficient c_g, with c_g = the geometrical thickness at a point on the diopter surface/geometrical thickness of the test plate. The uniformity of the optical coating on the diopter surface, as obtained in the evaporation geometry specified by the manufacturing technology, is numerically determined and it is used in the designing program for optical systems to evaluate the integral reflection and transmission factors of the coating. One can determine the sag of the intersection point of the ray with the diopter surface, the non-uniformity corresponding to the sag and can scale the geometrical thickness of the coating according to the non-uniformity. The reflection and transmission factors R_s, R_p, T_s, T_p as well as the phase changes in reflection φ_s^r, φ_s^r and transmission φ_s^t, φ_s^t for the scaled coating are determined with the same software functions as in the case of the designing program for optical coatings. One can determine the total intensities of the reflected or transmitted rays by means of which we can calculate the integral reflection and transmission factors of the coating on diopter i.

Software functions for the analysis of the spectral response of optical coatings in optical systems have been included in the designing programs for optical systems WINOPTIC V2.0[12]. The optical coatings are designed (including uniformity determination) with the program START V6.1[13]. The two programs share common software functions for the analysis of the optical coatings.

4. Example

The optical system chosen for analysis is simple and consists of a BK7 Shott-Mainz plane-convex lens with a radius of 62 mm, a 90 mm diameter and a 30 mm thickness at center. The object point P_o is at 100 mm from the vertex of the first diopter. The optical system is shown in Fig. 2.

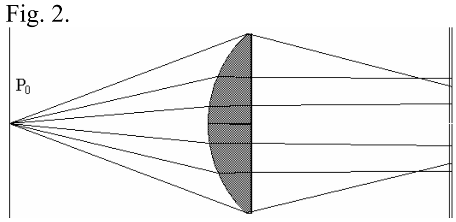

Fig. 2 Simple lens

The lens parameters were selected to prove the influence of non-uniformity of the spectral response of the optical coating. We consider that on both surfaces of the lens there is an antireflection optical coating with the following structure:

M/1.201L 0.750 H 0.361 L, 0.459H/S

Where L - MgF_2, quarter-wave optical thickness at 500 nm;
H – ZrO_2, quarter-wave optical thickness at 500 nm;
M – incidence medium: air, S – substrate: BK 7.

The spectral reflectance at normal incidence is shown in Fig. 4, curve 1.
We assume that the optical coating is produced in a vacuum installation where the evaporation geometry is as shown in Fig. 3 (e.g. the lens is put on a spherical, pyramidal or conical workholder).

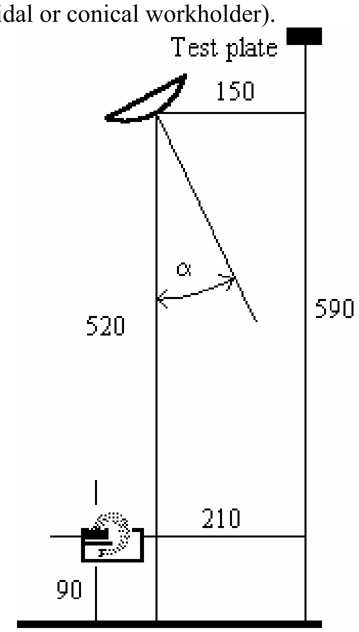

Fig. 3 The geometry of evaporation. Dimensions are in mm.

The uniformities of the optical coatings produced on the surface of the first diopter for three values of angle α are shown in Fig. 4 (we took a small area source, with polar distribution of vapor intensitie described by cosθ, θ being the angle with the normal to the source). In the following analysis, let us have the uniformity for $\alpha = 15^o$. The geometrical coefficient is considered symmetrical to the optical axis of the lens and depends linearly on the sag. In the following calculations the designed optical coating is considered to be produced (the scaling factor of the geometrical thickness for coating is $f_s = 1$) in the vertex of the diopter (where the maximum geometrical coefficient $c_g = 1.14$).

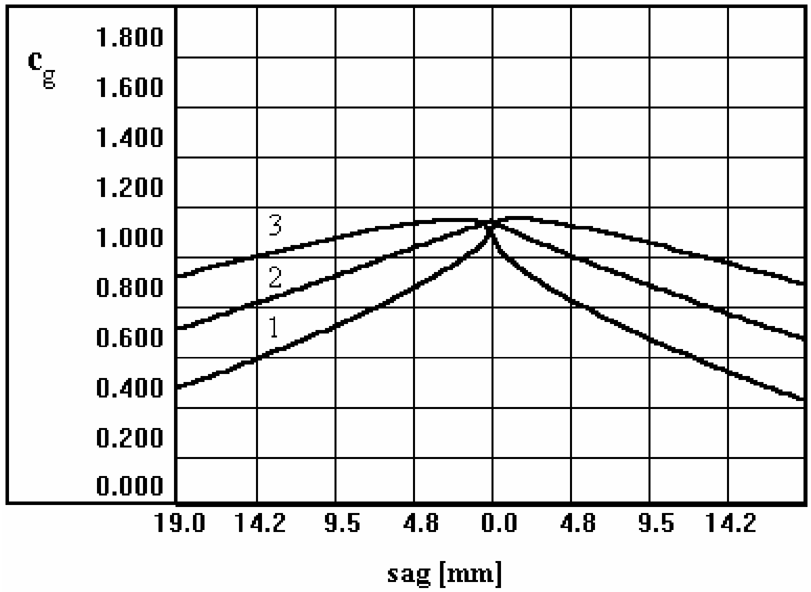

Fig. 4. The geometrical coefficient distribution on the diopter surface:
1 - $\alpha=0^o$; 2- $\alpha=15^o$; 3 - $\alpha=30^o$

For the other points on the diopter surface, the scaling factor is $f_s = c_g/1.14$. In Fig. 5, curves 3 and 4 represent the integral reflection factors of the first diopter coating for lens apertures of 30^o and 40^o. In Fig. 5, curve 2 represents the integral reflection factor for a uniform coating at a maximum aperture (the incidence angle of the rays generated for a maximum aperture on coating is 66^o). The object point P_o is considered to be a source of unpolarized light with a constant spectral radiance. As can be noticed, the integral reflection factor of a non-uniform coating for a maximum aperture of the lens is strongly changed related to the theoretical one calculated for a normal incidence.

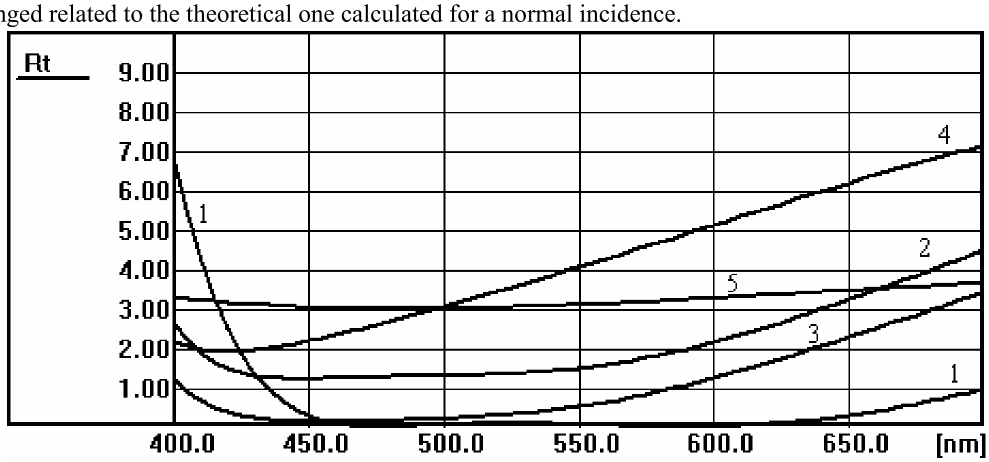

Fig. 5. Reflectance diopter 1. 1-normal incidence; 2 – uniform coating, aperture 40^o;
3 - aperture 30^o; 4 - aperture 40^o; 5 – antireflection M_gF_2, λ_o = 650nm, aperture 40^o.

Suppose that on the first diopter a simple layer antireflection coating of MgF_2, with an optical thickness nd = 650/4 nm, in the same geometry of evaporation (the same non-uniformity) is produced. The spectral response of the coating for a maximum aperture is shown in Fig. 5, curve 5. It is noticed that for a maximal aperture, the MgF_2 antireflection coating is superior to the multilayer antireflection coating.

5. Optimization of optical coating in the optical system

The above mentioned analysis, has shown that the spectral response of the optical coating on the first diopter is strongly modified. The question is what must be done to have a low reflection factor for the first diopter. The simplest solution is to replace the multilayer antireflection coating by a MgF_2, single layer coating, for instance. Another solution is to select some other geometry of evaporation (a planetary system for example) that should provide either a better uniformity or a specified non-uniformity. The spectral response of a coating when the coating is uniform is shown in Fig. 5, curve 2. When the geometry of evaporation is not changed, the solution is to optimize or redesign the optical coating taking into account the particular manufacturing conditions (uniformity) and the working conditions in that particular optical system. The optimization of optical coatings is discussed in Refs. 14-21. In all methods of optimization presented, the optical coatings have as targets the spectral parameters of coatings for various wavelengths, parameters determined for given incidences and polarization states.

As optical coatings are optimized by means of the designing programs for optical coatings and the spectral response is analyzed by means of the designing programs for optical systems, to optimize a coating having as targets the spectral response of a coating on a diopter in a particular optical system, a conversation must be established between the two programs, to allow for the data and commands exchange. The optimization process undergoes the following stages. The designing program for optical coatings communicates to the designing program for optical systems the coating to be optimized and the optimization targets. The designing program for optical systems evaluates the merit function and communicates it to the designing program for optical coatings, which accordingly modifies the coating parameters to attain an optimum, depending on the merit function evolution. The process is repeated until the merit function reaches an extreme (either minimum or maximum). With this optimization procedure the two applications can maintain their distinct identity, thus avoiding to mix-up in a single program software functions for the design of optical systems and coatings. At the same time this mechanism ensures the optimization of optical coatings having as targets any parameter that characterizes the optical system (the optical system may include light sources and detectors), and which depends on the optimized coating parameters. The conversations between programs provide data and commands exchange whenever the coating is modified under the designing program for optical coating (editing, optimization, photometric control process simulation with errors, etc.).

Acording to DDE Microsoft protocol, a conversation has been established between the two Win32 applications, STRAT V6.1 and WINOPTIC V2.0, by which data and commands exchange is achieved (STRAT V6.1 - client, WINOPTIC V2.0 -server). In the process of optimization, since the gradient can no longer be analytically determined, the Rosenbrock method[19] is used. The starting solution for optimization is that used in the above analysis. The optimization targets are the integral reflection factors for the first diopter, the lens having the maximal aperture and the non-uniformity specified above. The targets are in the spectral range of 450 – 650 nm, at intervals of 10 nm. The minimized merit function is:

$$F = \sum_i w_i Q_i$$

where $w_i = 1$ and Q_i are the integral reflection factors of the diopter one at the wavelengths where targets are defined. The optimized uniform coating that has been obtained has the following structure:

M/ 1.297L 1.056H 0.253L 0.595H / S

with the integral reflection factor is shown in Fig. 6, curve 2. Curve 3 represents the reflection factor for the coating at a normal incidence. The optimized non-uniform coating (non-uniformity specified in Fig. 3, α=15^o), has the following structure:

M/ 1.622L 1.697H 0.523L 0.544H - S

with the integral reflection factor shown in Fig. 7, curve 2. Curve 3 represents the reflection factor for the coating at a normal incidence. It is noticed that in order to compensate for the influence of the large incidences and the decrease in the geometrical thickness with the increase in sag, the optimized solutions have the spectral range with a low reflection factor at normal incidence shifted to the long wavelengths (the solutions are not necessarily obtained by scaling the geometrical thickness by a supra-unit factor). The optimized solutions do not have a spectacular improvement of the spectral response. It is difficult to find a coating solution that, for the above requested criteria, should not significantly modify its spectral response.

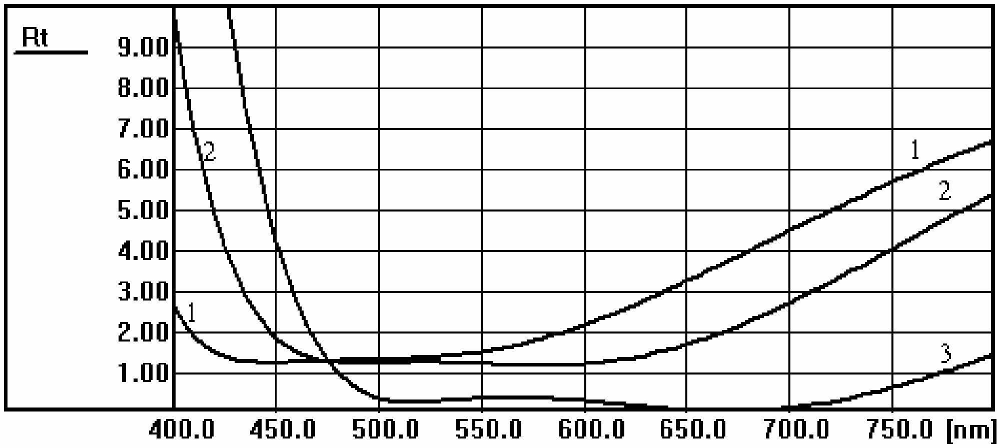

Fig. 6. Reflectance diopter 1, uniform optical coating: 1 – non-optimized; 2- optimized; 3 – optical coating , normal incidence

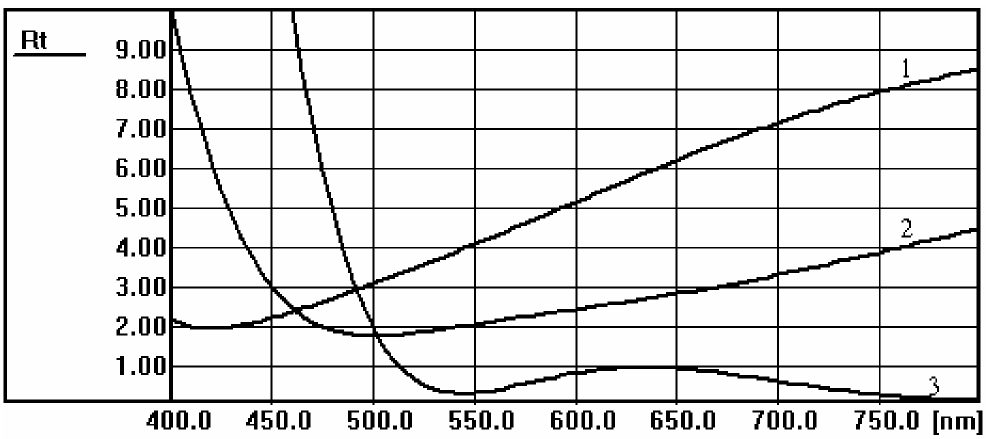

Fig. 7. Reflectance diopter 1, non-uniform optical coating : 1 – non-optimized; 2- optimized; 3 – optical coating , normal incidence

6. Conclusions

In view of the foregoing we may draw the conclusion that the specification of an optical coating for a certain diopter in a particular optical system can only be made after the spectral response of the coating in that optical system is analyzed. The analysis must also consider the uniformity of the optical coating obtained on the diopter surface, in the evaporation geometry existing in the manufacturing process. When the spectral response of the selected coating is not as expected, the coating must be optimized having as targets the spectral responses of the coating in that particular optical system, for specified wavelengths. Optimization is done by establishing a conversation between the designing programs for optical coatings and systems, a conversation that provides the data and commands exchange between the two programs.

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